

The scalar glueball and the loss of conformality

T. Nunes da Silva^a, E. Pallante^{a,*}, L. Robbroek^a

^a*Van Swinderen Institute for Particle Physics and Gravity, Nijenborgh 4, 9747 AG, Groningen, The Netherlands*

Abstract

We present a theoretical analysis of the behaviour of the anomalous dimension γ_G of the scalar glueball operator $G_{\mu\nu}^a G_{\mu\nu}^a$ at the non-trivial infrared fixed point inside the conformal window of a non-Abelian gauge theory. By virtue of the non-renormalisability of the energy-momentum tensor, γ_G is determined by the derivative of the gauge coupling beta function at the fixed point. We therefore predict γ_G from the perturbative beta-function to 2, 3, 4 loops, and show that its flavour number (N_f) dependence along the infrared fixed-point line is opposite to that implied by an ultraviolet-infrared fixed-point merging mechanism at the lower edge of the conformal window. A nonperturbative determination of γ_G can thus establish the mechanism for the loss of conformality and the fate of the preconformal phase. We also discuss large- N massless QCD in the Veneziano limit in this context.

Keywords: Non-Abelian gauge theories, QCD, conformal symmetry, conformal window

1. Introduction

In Quantum Chromodynamics (QCD) with a few massless fermions conformal symmetry is lost in a highly non trivial way. One single breaking phenomenon manifests itself in two forms: asymptotic freedom and confinement¹ — one cannot exist without the other. The recently proposed solution [1] for the scalar glueball two-point function in the 't Hooft limit of large- N QCD entails this property.

A new family of gauge theories is known to arise from QCD, for a sufficiently large number N_f of massless fermions in the fundamental representation, as well as in other representations of the gauge group. Such family is called the conformal window, ranging from N_f^c where the zero-temperature theory deconfines and chiral symmetry is restored, to N_f^{UV} where ultraviolet freedom is lost. Theories for $N_f^c < N_f < N_f^{UV}$ have a non-trivial, i.e., interacting infrared fixed point (IRFP) where they are conformal. A conformal window also arises in supersymmetric versions of non-Abelian gauge theories [2]. Above the conformal window $N_f > N_f^{UV}$, infrared freedom leads to the possibility of realising theories with an additional non-trivial ultraviolet fixed point (UVFP) [3]. For these reasons, all theories with $N_f > N_f^c$ may lead to new paradigms for particle dynamics beyond the standard model.

Just below the conformal window $N_f \lesssim N_f^c$, the phenomenologically interesting possibility of a preconformal

behaviour characterised by a walking, i.e., slow-running² gauge coupling has been proposed [4, 5]. Theories with a preconformal behaviour would not differ from QCD as far as their fixed point structure is concerned, i.e., they must be confining and asymptotically free³. The preconformal behaviour is directly related to the nature of the mechanism that opens the conformal window at N_f^c , and it should in some way affect the evolution from the UV to the IR of observables. It has been shown that a conformal phase transition [6–8] — equivalently a Berezinskii-Kosterlitz-Thouless (BKT) phase transition in two-dimensional spin systems [9–11] — leads to the walking phenomenon for $N_f \lesssim N_f^c$, and the associated preconformal behaviour of physical observables known as Miransky scaling or BKT scaling [6–11]. Interestingly, it was observed in [12] that a gauge coupling beta function leading to the loss of conformality at N_f^c via the merging of a pair of (UV and IR) fixed points is a simple way to realise this preconformal scaling. An interesting and still open question is if a conformal phase transition with its preconformal scaling can be realised without the need for a UV-IR fixed point merging mechanism at N_f^c . Ultimately, is a conformal phase transition realised at all in QCD? A phase transition of another nature at N_f^c , for example a first order one [13], is allowed and it would not lead to precursor effects.

In this letter we show that the anomalous dimension of the scalar glueball operator is a powerful probe of the mechanism for the loss of conformality at N_f^c ; its N_f dependence along the IRFP line inside the conformal win-

*Corresponding author

Email addresses: t.j.nunes@rug.nl (T. Nunes da Silva), e.pallante@rug.nl (E. Pallante), l.l.robboek@student.rug.nl (L. Robbroek)

¹Confinement is understood, in this context, as the existence of a mass-gap, i.e., a non-zero glueball mass in quenched QCD.

²At least on a finite energy range $[\mu_{IR}, \mu_{UV}]$.

³In other words, no phase transition is expected to occur between QCD and preconformal theories with $N_f \lesssim N_f^c$ at zero temperature.

dow reveals if a UV-IR fixed point merging does or does not occur. The following section 2 provides the anomalous dimension of the scalar glueball operator in terms of the beta function of the gauge coupling at a fixed point; this exact formula is valid for a non-Abelian gauge theory with fermions in any representation. In section 3, by means of this simple formula, we derive the scalar glueball anomalous dimension in perturbation theory, along the IRFP line for $N_f > N_f^c$ fermions in the fundamental representation, we compare with large- N predictions and discuss analogies and differences between QCD and supersymmetric QCD. We then show in section 4 how the prediction of a UV-IR fixed-point merging mechanism differs from the previous results. Inspired by the recent bounds on the location of the lower edge of the conformal window of QCD [14], we can now critically compare the implications of perturbative arguments with those of other proposed scenarios for the conformal window of QCD. We discuss this and prospects in the concluding section.

2. The scalar glueball operator and its anomalous dimension

It is well known that the anomalous dimension of the scalar glueball operator $\text{Tr}(G^2) \equiv G_{\mu\nu}^a G^{a\mu\nu}$ is constrained by the trace anomaly, i.e., the nonzero contribution to the trace of the energy-momentum tensor. The trace anomaly of QCD that enters the matrix elements of renormalised gauge invariant operators is ⁴

$$T_\mu^\mu = \frac{\beta(\alpha)}{16\pi\alpha^2} \text{Tr}(G^2) + \text{fermion mass contribution}, \quad (1)$$

with the beta function

$$\beta(\alpha) \equiv \frac{d\alpha(\mu)}{dt}, \quad t = \ln \mu, \quad \alpha \equiv \frac{g^2}{4\pi}. \quad (2)$$

The dimension of a quantum operator O is dictated by the scaling equation

$$\frac{dO}{d \ln \mu} = d_O O \quad O(\mu) \sim \mu^{d_O}, \quad (3)$$

$d_O = d_c + \gamma_O$, with canonical dimension d_c and anomalous dimension γ_O . The nonrenormalisation of T_μ^μ implies that it scales classically, i.e., $d_{T_\mu^\mu} = 4$ in four dimensions, and the scaling equation (3) applied to equation (1) gives for $\text{Tr}(G^2)$

$$d_G = 4 - \beta'(\alpha) + \frac{2}{\alpha}\beta(\alpha), \quad (4)$$

with $\beta'(\alpha)$ the derivative of the beta function with respect to α . At a fixed point, $\beta(\alpha_*) = 0$ and the anomalous dimension

$$\gamma_G = -\beta'(\alpha_*) \quad (5)$$

is a physical property of the system, renormalisation scheme independent.

⁴Here we are not interested in the most general expression, which also involves gauge-fixing and EoM operators, see [15–17].

3. Perturbative Results

It is instructive to determine γ_G in perturbation theory, where the beta function is known to a given loop order inside the conformal window. Later we compare this result with the Veneziano limit of large- N QCD ($N \rightarrow \infty$, $N_f/N = \text{const}$) [18]. The perturbative beta function of equation (2) can be expressed as a series

$$\beta(\alpha) = -2\alpha \sum_{l=1}^{\infty} b_l \alpha^l = -2\alpha \sum_{l=1}^{\infty} \bar{b}_l \alpha^l. \quad (6)$$

The quantities a and α

$$a \equiv \frac{g^2}{16\pi^2} = \frac{\alpha}{4\pi} \quad (7)$$

can interchangeably be used, times appropriate group invariants, as the expansion parameter and l denotes the number of loops involved in the calculation of b_l and $\bar{b}_l = b_l/(4\pi)^l$. From now on we use the coefficients \bar{b}_l , also used in [19, 20] for the numerical analysis, while b_l are used in [21–23]. The coefficients \bar{b}_1 and \bar{b}_2 are universal [24–27] and given by

$$\begin{aligned} \bar{b}_1 &= \frac{1}{3(4\pi)}(11C_A - 4T_f N_f) \\ \bar{b}_2 &= \frac{1}{3(4\pi)^2} [34C_A^2 - 4(5C_A + 3C_f)T_f N_f], \end{aligned} \quad (8)$$

here written in terms of the quadratic Casimir invariants $C_f \equiv C_2(R)$ and $C_A \equiv C_2(G)$, for, respectively, the representation R to which the N_f fermions belong and the adjoint representation. The quantity $T_f \equiv T(R)$ is the trace invariant for the representation R .

Coefficients of higher order are scheme-dependent [28, 29] and have been calculated up to four-loop order in the \overline{MS} scheme [21–23]. In Table 1 we list a subset of their values, for the $SU(N=3)$ theory with $6 \leq N_f \leq 12$ Dirac fermions in the fundamental representation: $C_A = N$, $C_f = (N^2 - 1)/(2N)$, and $T_f = 1/2$. The derivative

N_f	\bar{b}_1	\bar{b}_2	\bar{b}_3	\bar{b}_4
6	0.557	0.165	-0.0164	0.0991
7	0.504	0.0844	-0.118	0.0394
8	0.451	0.00422	-0.213	0.0147
9	0.398	-0.0760	-0.303	0.0253
10	0.345	-0.156	-0.386	0.0717
11	0.292	-0.236	-0.463	0.154
12	0.239	-0.317	-0.534	0.273

Table 1: The l -loop beta function coefficients \bar{b}_l , $l = 1, \dots, 4$ defined in equation (6), for the $SU(N=3)$ gauge theory with N_f Dirac fermions in the fundamental representation. Coefficients $\bar{b}_{1,2}$ from equation (8), $\bar{b}_{3,4}$ from $b_{3,4}$ in [22, 23].

$\beta'(\alpha)$ with respect to α can also be expressed in terms of

N_f	$\alpha_{\text{IR},2}$	$\beta'(\alpha_{\text{IR},2})$	$\alpha_{\text{IR},3}$	$\beta'(\alpha_{\text{IR},3})$	$\alpha_{\text{IR},4}$	$\beta'(\alpha_{\text{IR},4})$	$\alpha_{\text{UV},4}$
6	—	—	12.7	81.7	—	—	—
7	—	—	2.46	5.97	—	—	—
8	—	—	1.46	2.66	1.55	2.65	14.4
9	5.24	4.17	1.03	1.475	1.07	1.46	12.1
10	2.21	1.52	0.764	0.872	0.815	0.853	5.62
11	1.23	0.720	0.5785	0.517	0.626	0.498	3.29
12	0.754	0.360	0.435	0.2955	0.470	0.282	2.295

Table 2: Infrared zeros $\alpha_{\text{IR},n}$ at n -loop order, $n = 2, 3, 4$, for the $\text{SU}(N = 3)$ beta function with $N_f = 6, \dots, 12$ Dirac fermions in the fundamental representation. At four loops, the negative zero is not listed here, $\alpha_{\text{IR},4}$ is the positive one closest to the origin, and a third zero $\alpha_{\text{UV},4}$ occurs farther from the origin (possibly an artefact of the perturbative expansion).

the coefficients \bar{b}_l

$$\beta'(\alpha) = -2 \sum_{l=1}^{\infty} (l+1) b_l \alpha^l = -2 \sum_{l=1}^{\infty} (l+1) \bar{b}_l \alpha^l. \quad (9)$$

The value α_{IR} of the IRFP coupling is one root of the equation $\beta(\alpha) = 0^5$. At two loops $\alpha_{\text{IR},2} = -\bar{b}_1/\bar{b}_2$, and $\beta'(\alpha_{\text{IR},2}) = -2\bar{b}_1^2/\bar{b}_2 = -2b_1^2/b_2$. At four loops one has a cubic equation with three zeros, one of which is negative [30]. The numerical results for the case of interest are summarised in Table 2, and agree with those reported in [19, 20].

In all cases, the derivative $\beta'(\alpha_{\text{IR}})$ is positive and increases along the IRFP line for decreasing N_f . The disappearance of the zero occurs for $N_f > 8$ at two loops, while it shifts to lower N_f at three and four loops, suggesting a lower endpoint of the conformal window in the range $7 < N_f < 8$ at four loops—provided the zero can be taken as sufficient condition. Note that at two loops the disappearance of the zero is determined by the change of sign of \bar{b}_2 , implying that the fixed point disappears at infinite coupling $\alpha_{\text{IR},2} \rightarrow \infty$. This behaviour, however, is likely to be an artefact of the truncated perturbative expansion; the same singularity occurs in $\beta'(\alpha_{\text{IR},2})$.

It is most interesting to compare these results with the implications of the exact beta function derived in [18] for the large- N massless limit of QCD with N_f fundamental fermions in the Veneziano limit ($N_f, N \rightarrow \infty$, $N_f/N = \text{const}$), which manifests salient analogies and differences with the exact beta function of SQCD [31–33]. From inspection of the beta function [18]⁶

$$\beta(g) = \frac{dg}{d \ln \mu} = \left(-\frac{g^3}{16\pi^2} \right) \frac{4\pi\bar{b}_1 - N(\partial \ln Z' / \partial \ln \mu) + N_f \gamma_m(g)}{1 - N(g^2/4\pi^2)}, \quad (10)$$

⁵Note that the zero of the beta function is in general a necessary, but not sufficient condition for the existence of a stable IRFP.

⁶Equation (10) is written in terms of the canonical coupling $g_c = \sqrt{N}g$ in [18].

with the anomalous dimension factor

$$\frac{\partial \ln Z'}{\partial \ln \mu} = \frac{5}{24\pi^2} N g^2 \left(1 - \frac{2N_f}{5N} \right) + \dots \quad (11)$$

and the fermion mass anomalous dimension

$$\gamma_m = -\frac{9}{3(4\pi)^2} \frac{N^2 - 1}{N} g^2 + \dots \quad (12)$$

both of $O(Ng^2)$, and the comparison with the NSVZ beta function of SQCD [31–33], one concludes that the absence of supersymmetry generates the new anomalous dimension contribution in Z' for QCD.

The fate of a zero of the beta function, possibly associated to a stable fixed point, will depend on the numerator and denominator of equation (10): the pole generates a cusp in the RG flow of g , at $Ng^2 = 4\pi^2$, unless the numerator has a zero before the pole is hit. And a zero associated to an IRFP inside the conformal window must be shown to be renormalisation scheme independent. Remarkably, this has been shown to be true for the beta function in equation (10) at the lower edge of the conformal window, located at $N_f/N = 5/2$ where the stability of the glueball kinetic term is lost [18].

A fundamental difference between SQCD and QCD is the presence in SQCD of a phase just below the lower edge, for $N + 2 \leq N_f \leq 3N/2$, where the only description that makes physical sense is in terms of the dual variables that describe a free non-Abelian magnetic phase [2]; the original electric theory is infinitely coupled. The fact that the IR fixed point disappears at infinity $g_{\text{IR}} \rightarrow \infty$ at the lower edge of the conformal window, or equivalently the presence of the free magnetic phase, can coexist with the occurrence of a cusp in the RG flow of the gauge coupling. This occurs in SQCD when $\gamma_m(g^2 = 8\pi^2/N) \geq 1 - 3N/N_f$ [2], for which the pole of the beta function is hit before the zero of the numerator.

The absence of the same phase in QCD calls instead for a differentiable flow, thus without cusps, across and below the lower edge of the conformal window. It is rewarding that the beta function in [18] can realise this property. It also supports that the lower edge singularity for $\bar{b}_2 = 0$ of the two-loop beta function arises as an artefact of truncated perturbation theory, and that the IR fixed point

does not disappear at infinity at the lower edge, nor it merges with a UV fixed point. In this context, the $N_f = 0$ case is instructive. In [34] a renormalisation scheme for the large- N Yang-Mills exact beta function has been constructed, where the canonical coupling is shown to coincide with the physical effective charge g_{phys} entering the static inter-quark potential

$$V(r) = const + \sigma r - \frac{g_{phys}^2(1/r)}{4\pi r}, \quad (13)$$

with nonzero string tension σ . The effective charge g_{phys} in the Coulomb potential is observed to saturate to a constant at large distances in lattice $SU(3)$ Yang-Mills [35] in agreement with the effective bosonic string theory prediction [36], and it saturates according to the large- N beta function of [34]. In other words: the beta function of g_{phys} develops a zero, while conformal symmetry remains broken due to the linear confining contribution to the potential (non-zero string tension) dominating the large distance behaviour. Provided a RG transformation between the canonical coupling and the effective charge exists, we should not expect a qualitatively different behaviour for $N_f \neq 0$.

Coming back to γ_G , since the large- N beta function in [18] reproduces the two-loop one up to $O(1/N^2)$ contributions, the predicted γ_G in this case should also reproduce the two-loop result up to $O(1/N^2)$; on the other hand, its complete expression can be expected to remove the two-loop singularity for $\bar{b}_2 = 0$ at the lower edge of the conformal window. In summary, perturbation theory as well as the large- N solution predict an increasing magnitude of the anomalous dimension $|\gamma_G| = |\beta'(\alpha_*)|$ for $N_f \searrow N_f^c$.

4. Merging of UV and IR fixed points.

We show that the behaviour discussed above is opposite to the one implied by a UV-IR fixed point merging mechanism at N_f^c . In this scenario we can assume without loss of generality that close to N_f^c , where the merging would occur, we have [12]:

$$\beta(\alpha, N_f) = (N_f - N_f^c) - (\alpha - \alpha_c)^2. \quad (14)$$

For completeness, we recall the properties of this beta function first discussed in [12]. It has zeros at $\alpha_{\pm} = \alpha_c \pm \sqrt{N_f - N_f^c}$. As shown in Fig. 1 (top), α_{\pm} are distinct and real for $N_f > N_f^c$, they coincide $\alpha_+ = \alpha_- = \alpha_c$ for $N_f = N_f^c$ (the UV-IR fixed point merging) and become complex for $N_f < N_f^c$, thus leading to the disappearance of the conformal window. Below the conformal window, for N_f sufficiently close to N_f^c , the gauge coupling increases towards the IR from an initial value α_{UV} at some UV scale μ_{UV} , as qualitatively shown in Fig. 1 (bottom). In the region where the beta function is small, the coupling will “walk” until it blows up at some IR scale μ_{IR} . The latter is obtained by integrating the beta function on $[\mu_{UV}, \mu_{IR}]$, it defines the longest correlation length

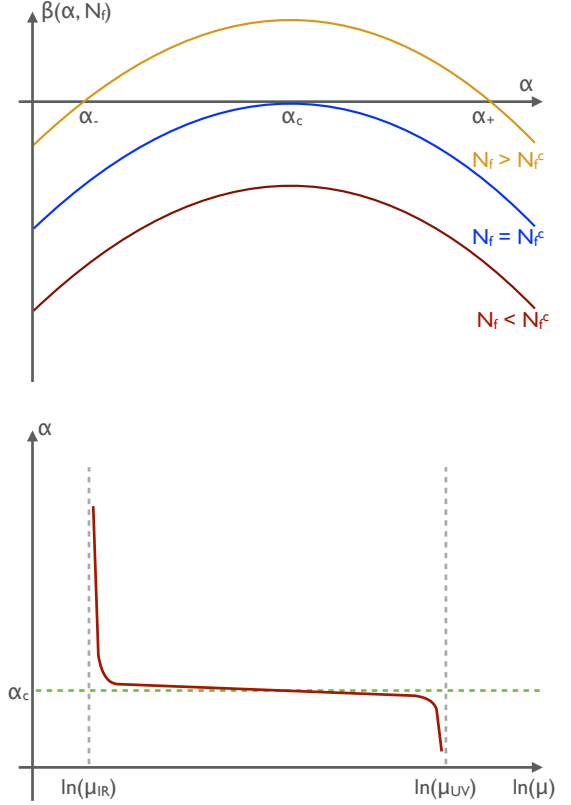


Figure 1: (top) The beta function $\beta(\alpha, N_f)$ of equation 14 for decreasing N_f , top to bottom: for $N_f > N_f^c$ there is a pair of fixed points at α_- (IRFP) and α_+ (UVFP). They merge at α_c for $N_f = N_f^c$ and disappear for $N_f < N_f^c$. (bottom) Sketch of the renormalisation group flow of the coupling α as a function of the energy scale $\ln \mu$ just below the conformal window for $N_f \lesssim N_f^c$.

in the system and realises BKT/Miransky scaling as long as $|\alpha_{IR,UV} - \alpha_c| \gg |N_f - N_f^c|$ ($\alpha_{UV} < \alpha_c < \alpha_{IR}$)

$$\mu_{IR} = \mu_{UV} \exp \left[\int_{\alpha_{UV}}^{\alpha_{IR}} \frac{d\alpha}{\beta(\alpha, N_f)} \right] \simeq \exp \left[- \frac{\pi}{\sqrt{(N_f^c - N_f)}} \right]. \quad (15)$$

Here, we first observe that this beta function develops a local maximum at $N_f = N_f^c$, i.e., $\beta'(\alpha_c) = 0$, with decreasing $|\beta'|$ along the IRFP line for decreasing $N_f \searrow N_f^c$. In other words, oppositely to what predicted by perturbation theory, the magnitude of the scalar glueball anomalous dimension γ_G is predicted to decrease for decreasing N_f inside the conformal window, and vanishes at its lower edge. This may pose some problems. It is rather counterintuitive to have the anomalous dimension of the scalar glueball operator, a probe of confinement, that diminishes while the theory is approaching the confined phase and the screening of the gauge force diminishes. On the contrary, perturbation theory predicts what we do expect. Secondly, one

has to guarantee that the beta function of equation (14) smoothly metamorphoses into the perturbative QCD beta function for any $N_f < N_f^c$ and $\alpha \ll \alpha_c$; another non-trivial test to be passed is that the correct UV behaviour of correlation functions dictated by asymptotic freedom must be reproduced. Finally, of interest beyond QCD remains the question if, and if so how, Miransky/BKT scaling can occur without a beta function with a UV-IR fixed point merging.

5. Prospects

We have shown that the scalar glueball operator, a probe of confinement, carries information on the nature of the lower edge of the conformal window and the zero temperature phase that precedes it. In particular, perturbation theory and the large- N limit of QCD predict that the magnitude of its anomalous dimension increases inside the conformal window along the IRFP line for $N_f \searrow N_f^c$, while it decreases in the presence of a UV-IR fixed point merging mechanism and vanishes at the lower edge; we observe that it is more difficult to reconcile the latter behaviour, which is associated to Miransky/BKT scaling, with the fact that theories do confine below the conformal window.

Conversely, the trend of the anomalous dimension γ_G predicted by perturbation theory, as well as by the large- N limit of QCD, is rather intuitive. These results together with the predicted location of the lower edge of the conformal window of QCD, see Table 2 and [18], offer a picture quite consistent with the recently determined bound on the lower edge $6 < N_f^c < 8$ [14] based on a lattice study.

This analysis also suggests that a nonperturbative determination of γ_G for varying N_f inside the conformal window would finally establish if a UV-IR fixed point merging is realised or not in QCD. It would also provide a measure of how close the complete theory is to the predictions of perturbation theory and its large- N limit. This measure can be achieved on the lattice through the study of the two-point correlation function of the scalar glueball operator; such studies, however, are a notoriously difficult challenge [37]. We also recognise that the Wilson flow proposed in [38, 39] can be a useful tool in this context, in order to discriminate between a conformal and a confining behaviour in the theory formulated on a lattice [40]. Inside the conformal window, and in the deconfined high-temperature phase of QCD, the AdS/CFT correspondence [41] as well as conformal bootstrap can also offer valuable insights. Finally, an extension of this study to theories with different gauge groups and/or fermionic matter in higher-dimensional representations would also be instructive and it has relevance for phenomenology beyond the standard model.

Acknowledgments

We thank M. Bochicchio for valuable comments and discussions.

References

- [1] M. Bochicchio, Nucl. Phys. B875 (2013) 621.
- [2] N. Seiberg, Nucl. Phys. B435 (1995) 129.
- [3] D. F. Litim, F. Sannino, JHEP 12 (2014) 178.
- [4] T. Appelquist, D. Karabali, L. C. R. Wijewardhana, Phys. Rev. Lett. 57 (1986) 957.
- [5] K. Lane, M. V. Ramana, Phys. Rev. D44 (1991) 2678.
- [6] T. Appelquist, et al., Phys. Rev. D58 (1998) 105017.
- [7] V. A. Miransky, K. Yamawaki, Phys. Rev. D55 (1997) 5051.
- [8] V. Miransky, Nuovo Cim. A90 (1985) 149.
- [9] V. L. Berezinsky, Sov. Phys. JETP 32 (1971) 493.
- [10] V. L. Berezinsky, Sov. Phys. JETP 34 (1972) 610.
- [11] J. M. Kosterlitz, D. J. Thouless, J. Phys. C6 (1973) 1181.
- [12] D. B. Kaplan, et al., Phys. Rev. D80 (2009) 125005.
- [13] F. Sannino, Mod. Phys. Lett. A28 (2013) 1350127.
- [14] T. Nunes da Silva, E. Pallante, L. Robroek [arXiv:1506.06396](#).
- [15] N. Nielsen, Nucl. Phys. B120 (1977) 212.
- [16] J. C. Collins, A. Duncan, S. D. Joglekar, Phys. Rev. D16 (1977) 438.
- [17] S. L. Adler, J. C. Collins, A. Duncan, Phys. Rev. D15 (1977) 1712.
- [18] M. Bochicchio [arXiv:1312.1350](#).
- [19] R. Shrock, Phys. Rev. D87 (2013) 116007.
- [20] R. Shrock, Phys. Rev. D87 (2013) 105005.
- [21] O. Tarasov, A. Vladimirov, A. Y. Zharkov, Phys. Lett. B93 (1980) 429.
- [22] S. Larin, J. Vermaseren, Phys. Lett. B303 (1993) 334.
- [23] T. van Ritbergen, J. Vermaseren, S. Larin, Phys. Lett. B400 (1997) 379.
- [24] D. J. Gross, F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343.
- [25] H. D. Politzer, Phys. Rev. Lett. 30 (1973) 1346.
- [26] W. Caswell, Phys. Rev. Lett. 33 (1974) 244.
- [27] D. Jones, Nucl. Phys. B75 (1974) 531.
- [28] D. J. Gross, F. Wilczek, Phys. Rev. D8 (1973) 3633.
- [29] D. J. Gross, R. Balian, J. Zinn-Justin, Methods in Field Theory, Les Houches 1975, 1976.
- [30] T. A. Ryttov, R. Shrock, Phys. Rev. D83 (2011) 056011.
- [31] V. Novikov, et al., Nucl. Phys. B229 (1983) 381.
- [32] V. Novikov, et al., Phys. Lett. B166 (1986) 329.
- [33] M. Shifman, A. Vainshtein, Nucl. Phys. B277 (1986) 456.
- [34] M. Bochicchio, JHEP 0905 (2009) 116.
- [35] M. Lüscher, P. Weisz, JHEP 2002 (2002) 049.
- [36] M. Lüscher, P. Weisz, JHEP 2004 (2004) 014.
- [37] B. Lucini, PoS QCD-TNT-III (2013) 023.
- [38] M. Lüscher, JHEP 1008 (2010) 071.
- [39] R. Narayanan, H. Neuberger, JHEP 0603 (2006) 064.
- [40] E. Pallante, in: Proc. of the Sakata Memorial KMI Workshop “Origin of Mass and Strong Coupling Gauge Theories” (SCGT15), 2015. [arXiv:1509.00733](#).
- [41] O. Aharony, et al., Phys. Rept. 323 (2000) 183.